

MATHEMATICS

2012 HSC Course Assessment Task 3 (Trial Examination) June 21, 2012

General instructions

- Working time 3 hours.
 (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt all questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.

(SECTION I)

• Mark your answers on the answer sheet provided (numbered as page 9)

(SECTION II)

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER:		# BOOKLETS USED:
Class (please \checkmark)		
\bigcirc 12M2A – Mr Berry	○ 12M	I3C – Ms Ziaziaris
	○ 12M	I3D – Mr Lowe
	○ 12M	M3E – Mr Lam

Marker's use only.

QUESTION	1-10	11	12	13	14	15	16	Total
MARKS	10	15	15	15	15	15	15	100

Section I: Objective response

Mark your answers on the multiple choice sheet provided.

Marks

1. What is the value of |-8| - |10|?

1

(A) 2

(B) 1

(C) -1

(D) -2

What is the sum of the exterior angles of a polygon?

1

(A) 90°

(B) 180°

(C) 360°

(D) none of these

Which conditions make the quadratic $y = ax^2 + bx + c$ positive definite?

1

(A) $a < 0, \Delta < 0$ (B) $a < 0, \Delta > 0$ (C) $a > 0, \Delta < 0$ (D) $a > 0, \Delta > 0$

Which of the following is *not* a condition for congruent triangles?

1

(A) SSS

(B) AAA

(C) SAS

(D) AAS

What is 1.9926 to two significant figures?

1

(A) 2.0

(B) 1.9

(C) 2.09

(D) 2.01

Which of the following is the locus of a point that is equidistant from a fixed point and a fixed line?

1

(A) a parabola

(B) a hyperbola

(C) a circle

(D) an ellipse

7. Evaluate $\lim_{x \to \infty} \frac{3x^5 - 2x^2 + 7x - 3}{6x^5 - 3x + 7}$.

1

(A) ∞

(B) 0

(C) $\frac{1}{2}$

(D) 2

Which of the following conditions for $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ describe the slowing growth of a variable P?

1

(A) $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt} > 0$.

(C) $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt} < 0$.

(B) $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt} < 0$.

(D) $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt} > 0$.

If a > b, which of the following is always true?

1

(A) $a^2 > b^2$ (B) $\frac{1}{a} > \frac{1}{b}$ (C) -a > -b (D) $2^a > 2^b$

10. What is the exact value of b if the area beneath the curve $y = \frac{2}{x}$ between x = 11 and x = b (b > 1) is equal to 3 units²?

(A) $e^{\frac{3}{2}}$

(B) e^2

(C) $e^{\frac{5}{2}}$

(D) e^{3}

3

1

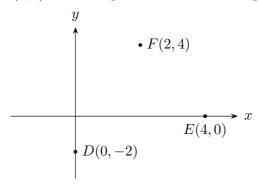
Section II: Short answer

Question 11 (15 Marks)

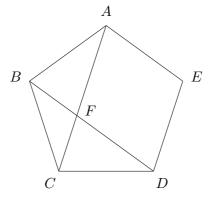
Commence a NEW page.

Marks

- (a) Solve the equation $x^2 + \frac{9}{x^2} = 10$.
- (b) D(0,-2), E(4,0) & F(2,4) are three points on the number plane.



- i. Calculate the length of the interval DF.
- ii. Calculate the gradient of DF.
- iii. Write the equation of the line DF in general form.
- iv. Calculate the perpendicular distance from E to the line DF.
- v. Calculate the area of $\triangle DEF$.
- (c) ABCDE is a regular pentagon. The diagonals AC and BD intersect at F.



Copy or trace this diagram into your writing booklet. By giving full reasons for your answer,

- i. Prove that $\angle ABC = 108^{\circ}$.
- ii. Find the size of $\angle BAC$.
- (d) Find the exact value of $3 \tan 210^{\circ} + 2 \sin 300^{\circ}$.

Question 12 (15 Marks) Commence a NEW page. Marks Find the values of p, p > 0 for which the roots of the equation $x^2 - px + p = 0$ (a) opposite in sign. 1 i. ii. $\mathbf{2}$ real $\mathbf{2}$ (b) Sketch the parabola with equation $(y-2)^2 = 2(x+2)$ Show the vertex of the parabola on your sketch. $\mathbf{2}$ Find the coordinates of the focus and the equation of the directrix of the parabola. The sum to n terms of a sequence of numbers is given by $S_n = 102n - 2n^2$. (c) Find an expression for T_n , the *n*-th term of the sequence. $\mathbf{2}$ What type of a sequence is this? 1 (d) Differentiate the following expressions: i. 1 ii. $3\cos 4x$ $\mathbf{2}$ iii. $\log_e(2x)$ $\mathbf{2}$ Question 13 (15 Marks) Commence a NEW page. Marks For the curve $y = x^3(4-x)$ (a) i. Find the stationary point(s) and determine their nature. 3 Find the point(s) of inflexion. $\mathbf{2}$ Draw a neat sketch of the curve showing the intercepts with the coordinate 3 axes, any stationary points and any point(s) of inflexion. Find f(x) if $f'(x) = 2x + \frac{1}{x^2}$ and the curve passes through the point (1,2). (b) 3

(c)

Find the primitive of

ii. $3\sec^2\frac{x}{3}$

 $\mathbf{2}$

 $\mathbf{2}$

 $\mathbf{2}$

3

1

Question 14 (15 Marks)

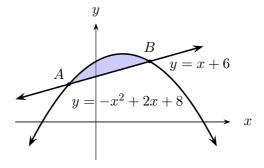
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Marks

(a) Use Simpson's Rule with five function values to evaluate $\int_1^3 f(x) dx$ given the following table:

Ī	x	1	1.5	2	2.5	3
	f(x)	0	3	5	2	1

(b) The diagram below shows the graphs of $y = -x^2 + 2x + 8$ and y = x + 6.



- i. Show that the x coordinate of A and B are x = -1 and x = 2 respectively.
- ii. Hence or otherwise, find the shaded area bounded by the curves and the straight line.
- (c) i. State the period and amplitude of $y = 3 \sin 2x$.
 - ii. Draw a neat sketch of $y = 3\sin 2x$, where $0 \le x \le 2\pi$.
 - iii. Hence or otherwise, state the number of solutions to the equation

$$3\sin 2x = \frac{2}{3}$$

within the domain $0 \le x \le 2\pi$.

(d) Find the angle subtended at the centre of the circle of sector with radius 4 cm and area 20 cm². Give your answer correct to the nearest degree.

Question 15 (15 Marks)

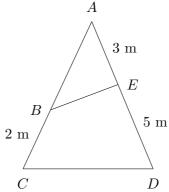
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Marks

 $8^x = 16^{x+1} \times 4^{-x}.$ Solve (a)

 $\mathbf{2}$

In this diagram, $\angle BCD + \angle BED = 180^{\circ}$. (b)



Prove that $\triangle ABE$ is similar to $\triangle ADC$.

3

3

Given that AE = 3 m, ED = 5 m and BC = 2 m, calculate the length of

i. Evaluate $\frac{d}{dx}(\log_e(\sin x))$. (c)

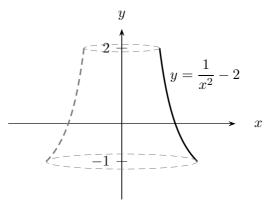
1

ii. Hence or otherwise, find $\int \cot x \, dx$.

4

 $\mathbf{2}$

(d) A liquor bottle is obtained by rotating about the y axis the part of the curve $y = \frac{1}{x^2} - 2$ between y = -1 and y = 2.



Find the exact volume of the bottle.

Examination continues overleaf...

1

3

4

1 2

 $\mathbf{2}$

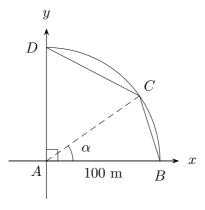
Question 16 (15 Marks)

(b)

Commence a NEW page.

Marks

(a) ABCD is a quadrilateral inscribed in a quarter of a circle centred at A with radius 100 m. The points B and D lie on the x and y axes and the point C moves on the circle such that $\angle CAB = \alpha$ as shown in the diagram below.



- i. Solve the equation $\sin(x+15^\circ) = \cos 24^\circ$.
- ii. Show that the area of the quadrilateral ABCD can be expressed as

$$A = 5000(\sin \alpha + \cos \alpha)$$

- iii. Show that the maximum area of this quadrilateral is $5000\sqrt{2}$ m².
 - 2
- ii. Consider the series $2e^x + 8e^{-x} + 32e^{-3x} + \cdots$

Sketch the curve $y = 4e^{-2x}$.

Consider the series $2e + 6e + 32e + \cdots$

 α) Show that this series is geometric.

- β) Find the values of x for which this series has a limiting sum.
- γ) Find the limiting sum of this series in terms of x.

End of paper.

STANDARD INTEGRALS

$$\int x^{n} dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x + C, \qquad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \qquad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + C, \qquad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C, \qquad a \neq 0$$

$$\int \sec^{2} ax dx = \frac{1}{a} \tan ax + C, \qquad a \neq 0$$

$$\int \sec^{2} ax dx = \frac{1}{a} \tan ax + C, \qquad a \neq 0$$

$$\int \frac{1}{a^{2} + x^{2}} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^{2} - x^{2}}} dx = \sin^{-1} \frac{x}{a} + C, \qquad a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^{2} - a^{2}}} dx = \ln \left(x + \sqrt{x^{2} - a^{2}} \right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^{2} + a^{2}}} dx = \ln \left(x + \sqrt{x^{2} + a^{2}} \right) + C$$

NOTE: $\ln x = \log_e x, x > 0$

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●" STUDENT NUMBER:

Class (please ✓)

 \bigcirc 12M2A – Mr Berry

- \bigcirc 12M3C Ms Ziaziaris
- \bigcirc 12M3D Mr Lowe
- \bigcirc 12M3E Mr Lam

- 1 (A) (B) (C) (D)
- $\mathbf{2}$ \mathbb{A} \mathbb{B} \mathbb{C} \mathbb{D}
- 3 (A) (B) (C) (D)
- 4 (A) (B) (C) (D)
- $\mathbf{5}$ \mathbf{A} \mathbf{B} \mathbf{C} \mathbf{D}
- 6 (A) (B) (C) (D)
- 7 ABCOD
- 8- (A) (B) (C) (D)
- 10 (A) (B) (C) (D)

Suggested Solutions

Section I

(Lowe) 1. (D) 2. (C) 3. (D) 4. (B) 5. (A) 6. (A) 7. (C) 8. (C) 9. (D) 10. (A)

Question 11 (Lowe)

- (a) (3 marks)
 - \checkmark [1] for quartic.
 - \checkmark [1] for final solutions.

$$x^{2}_{\times x^{2}} + \frac{9}{x^{2}} = 10_{\times x^{2}}$$

$$x^{4} + 9 = 10x^{2}$$

$$x^{4} - 10x^{2} + 9 = 0$$

$$(x^{2} - 9)(x^{2} - 1) = 0$$

$$\therefore x = \pm 1, \pm 3$$

(b) i. (1 mark)

$$DF = \sqrt{(2-0)^2 + (4-(-2))^2}$$
$$= \sqrt{2^2 + 6^2} = \sqrt{40}$$
$$= 2\sqrt{10}$$

(1 mark)

$$m_{DF} = \frac{6}{2} = 3$$

(1 mark)iii.

$$\frac{y-4}{x-2} = 3$$
$$y-4 = 3x-6$$
$$3x-y-2 = 0$$

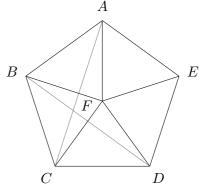
- iv. (2 marks)
 - for correctly recalling \checkmark [1] perpendicular dist formula.
 - \checkmark [1] for final answer.

$$d_{\perp} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$
$$= \frac{|3(4) + (-1)(0) - 2|}{\sqrt{3^2 + 1^2}}$$
$$= \frac{10}{\sqrt{10}} = \sqrt{10}$$

- v. (2 marks)
 - ✓ [1] for using parts (iii) & (iv)
 - \checkmark [1] for final answer.

$$A = \frac{1}{2} \times DF \times d_{\perp}$$
$$= \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10}$$
$$= 10 \text{ units}^{2}$$

- (c) i. (2 marks)
 - Divide pentagon five equilateral triangles.



- Apex angle of one of the triangles $\frac{360^{\circ}}{5} = 72^{\circ}.$
- Angle sum of the two base angles is thus

$$180^{\circ} - 72^{\circ} = 108^{\circ}$$

- ii. (1 mark)

 - △BAC is isosceles.
 ∴ ∠BAC = ^{180°-108°}/₂ = 36°.
- (d) (2 marks)

$$3\tan 210^{\circ} + 2\sin 300^{\circ}$$
$$= 3 \times \left(\frac{1}{\sqrt{3}}\right) + 2 \times \left(-\frac{\sqrt{3}}{2}\right)$$
$$= \frac{3}{\sqrt{3}} - \sqrt{3} = 0$$

Question 12 (Lowe)

(a) i. (1 mark) $x^2 - px + p = 0$ $\alpha = -\beta$ $\therefore \alpha + \beta = 0 = -\frac{b}{a} = p$

But as p > 0, therefore there are no real solutions.

ii. (2 marks)
$$\checkmark \quad [1] \text{ for } p \leq 0 \text{ or } p \geq 4.$$

$$\checkmark \quad [1] \text{ justify why } p \geq 4 \text{ only.}$$

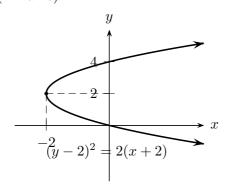
$$\Delta \ge 0$$

$$\therefore b^2 - 4ac = p^2 - 4p \ge 0$$

$$p(p-4) \ge 0$$

$$\therefore p \le 0 \text{ or } p \ge 4$$

But as p > 0, hence $p \ge 4$ only.



ii. (2 marks)

$$4a = 2$$

$$\therefore a = \frac{1}{2}$$

$$S\left(-\frac{3}{2}, 2\right)$$

Directrix is $x = -\frac{5}{2}$.

$$S_n = 102n - 2n^2$$

$$T_n = S_n - S_{n-1}$$

$$= 102n - 2n^2$$

$$- \left(102(n-1) - 2(n-1)^2\right)$$

$$= 102n - 2n^2$$

$$- \left(102n - 102 - 2(n^2 - 2n + 1)\right)$$

$$= 2n^2 + 102 + 2n^2 - 4n + 2$$

$$= 104 - 4n$$

$$T_1 = 104 - 4(1) = 100$$

 $T_2 = 104 - 4(2) = 96$
 $T_3 = 104 - 4(3) = 92$
 $T_3 - T_2 = T_2 - T_1$

Arithmetic sequence.

$$\frac{d}{dx}\left(2x^{-3}\right) = -6x^{-4}$$

$$\frac{d}{dx}(3\cos 4x) = -12\sin 4x$$

$$\frac{d}{dx}(\log_e 2x) = \frac{d}{dx}(\log_e 2 + \log_e x) = \frac{1}{x}$$

Question 13 (Berry)

(a) i. (3 marks)

$$y = 4x^{3} - x^{4} = x^{3}(4 - x)$$
$$\frac{dy}{dx} = 12x^{2} - 4x^{3}$$

Stationary pts occur when $\frac{dy}{dx} = 0$:

$$4x^2(3-x) = 0$$

$$\therefore x = 0.3$$

x		0		3	
$\frac{dy}{dx}$	+	0	+	0	_
y		, 0		27	

Hence (0,0) is a horizontal point of inflexion and (3,27) is a local maximum.

ii. (2 marks)

Points of inflexion occur when $\frac{d^2y}{dx^2} = 0$:

$$\frac{d^2y}{dx^2} = 24x - 12x^2 = 12x(2-x)$$

$$\therefore x = 0, 2$$

x		0		2	
d^2y	_	0	+	0	_
$\frac{d^2y}{dx^2}$			\smile		

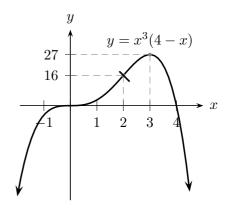
When x = 2,

$$y = x^{3}(4-x)\Big|_{x=2}$$
$$= 2^{3}(4-2) = 16$$

Hence points of inflexion occur at (0,0) and (2,16) as concavity changes at these two pts.

iii. (3 marks)

 \checkmark [-1] for each omission from requirements of the question, provided graph is correct.



b) (3 marks)

- \checkmark [1] for correct integral.
- \checkmark [1] for correct value of C.
- \checkmark [1] for final answer.

$$f(x) = \int (2x + x^{-2}) dx = x^{2} - x^{-1} + C$$
$$f(1) = 1 - 1^{-1} + C = 2$$
$$\therefore C = 2$$
$$\therefore f(x) = x^{2} - \frac{1}{x} + 2$$

(c) i. (2 marks)

 \checkmark [-1] if missing arbitrary constant.

$$\int x^{\frac{1}{3}} dx = \frac{3}{4} x^{\frac{4}{3}} + C$$

ii. (2 marks)

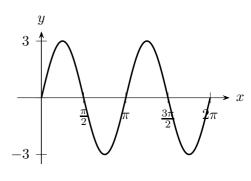
 $\sqrt{[-1]}$ if missing arbitrary constant.

$$\int 3\sec^2\frac{x}{3}\,dx = 9\tan\frac{x}{3} + C$$

Question 14 (Berry)

(a) (2 marks)

$$\int_{1}^{3} f(x) dx = \frac{h}{3} (y_0 + 4 \sum y_{\text{odd}} + 2 \sum y_{\text{even}} + y_{\ell})$$
$$= \frac{\frac{1}{2}}{3} (0 + 1 + 4(3 + 2) + 2(5))$$
$$= \frac{31}{6}$$



iii. (1 mark) 4 solutions.

(b) i. (2 marks)

$$\begin{cases} y = -x^2 + 2x + 8 \\ y = x + 6 \end{cases}$$

(d) (2 marks)

 \checkmark [1] for answer in radians.

 \checkmark [1] for answer in degrees.

Solve by equating,

$$-x^{2} + 2x + 8 = x + 6$$

$$x^{2} - x + 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$\therefore x = -1, 2$$

$$A = \frac{1}{2}r^2\theta$$
$$20 = \frac{1}{2} \times 4^2 \times \theta$$
$$\theta = \frac{5}{4} = \frac{5}{4} \times \frac{180^{\circ}}{\pi} \approx 71^{\circ}$$

ii. (3 marks)

$$A = \left| \int_{-1}^{2} (x^2 - x + 2) dx \right|$$

$$= \left| \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right]_{-1}^{2} \right|$$

$$= \left| -\frac{1}{3} (2^3 - (-1)^3) + \frac{1}{2} (2^2 - (-1)^2) + 2(2 - (-1)) \right|$$

$$= \left| -3 + \frac{3}{2} + 6 \right| = \frac{9}{2}$$

(c) i. (2 marks)

$$T = \frac{2\pi}{2} = \pi \qquad a = 3$$

ii. (3 marks)

 \checkmark [1] for shape.

√ [1] for correct period.

 \checkmark [1] for amplitude.

Question 15 (Ziaziaris)

(a) (2 marks)

 \checkmark [1] for resolving into powers of 2.

 \checkmark [1] for final answer.

$$8^{x} = 16^{x+1} \times 4^{-x}$$

$$(2^{3})^{x} = (2^{4})^{x+1} \times (2^{2})^{-x}$$

$$2^{3x} = 2^{4x+4} \times 2^{-2x}$$

$$2^{3x} = 2^{2x+4}$$

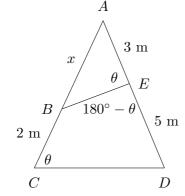
$$3x = 2x + 4$$

$$x = 4$$

(b) i. (3 marks)

 \checkmark [1] for each correct reason.

In $\triangle ABE$ and $\triangle ADC$



- $\angle CAD$ (common)
- Let $\angle BCD = \theta$. From the information,

$$\angle BED = 180^{\circ} - \theta$$

Hence $\angle AEB = \theta$ (supplementary), and $\angle ACD = \angle AEB$.

• $\angle ABE = \angle ADC$ (remaining \angle)

Hence $\triangle ABE \parallel \triangle ACD$ (equiangular)

- ii. (3 marks)
 - \checkmark [1] for ratio of lengths.
 - \checkmark [1] for setting up quadratic.
 - \checkmark [1] for final answer.

Let AB = x. As the ratio of the side lengths of corresponding sides

are equal,

$$\frac{AB}{AD} = \frac{AE}{AC}$$

$$\frac{x}{8} = \frac{3}{x+2}$$

$$x(x+2) = 24$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

$$\therefore x = 4, -6$$

As x > 0 (length), $\therefore x = 4$ only.

(c) i. (1 mark)

$$\frac{d}{dx}\left(\log_e\left(\sin x\right)\right) = \frac{\cos x}{\sin x}$$

ii. (2 marks)

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx$$
$$= \log_e(\sin x) + C$$

(d) (4 marks)

$$y = \frac{1}{x^2} - 2$$

$$y + 2 = \frac{1}{x^2}$$

$$x^2 = \frac{1}{y+2}$$

$$V = \pi \int_{-1}^2 x^2 \, dy = \pi \int_{-1}^2 \frac{dy}{y+2}$$

$$= \pi \left[\log_e(y+2) \right]_{-1}^2$$

$$= \pi \left(\log_e 4 - \log_e 1 \right)$$

$$= \pi \log_e 4$$

Question 16 (Lam)

i. (2 marks)

$$\sin(x+15^\circ) = \cos 24^\circ = \sin(90^\circ - 24^\circ)$$
$$x+15^\circ = 66^\circ$$
$$\therefore x = 51^\circ$$

ii. (3 marks)

$$A_{\triangle CAB} = \frac{1}{2} \times 100^{2} \sin \alpha = 5\,000 \sin \alpha \qquad \text{ii. } (\alpha) \qquad (1 \text{ mark})$$

$$2e^{x} + 8e^{-x} + 32e^{-3x} \cdots$$

$$A_{\triangle CAD} = \frac{1}{2} \times 100^{2} \sin \left(\frac{\pi}{2} - \alpha\right) = 5\,000 \cos \alpha$$

$$A_{ABCD} = 5\,000 \left(\sin \alpha + \cos \alpha\right) \qquad \frac{T_{2}}{T_{1}} = \frac{8e^{-x}}{2e^{x}} = 4e^{-\frac{x}{2}}$$

iii. (4 marks)

$$A_{ABCD} = 5\,000\,(\sin\alpha + \cos\alpha)$$
$$\therefore \frac{dA}{d\alpha} = 5\,000(\cos\alpha - \sin\alpha)$$

Stationary pts occur when $\frac{dA}{d\alpha} = 0$, i.e.

$$5000(\cos \alpha - \sin \alpha) = 0$$
$$\cos \alpha = \sin \alpha$$
$$\div \cos \alpha = \frac{1}{2}$$
$$\tan \alpha = 1$$
$$\therefore \alpha = \frac{\pi}{4}$$

α	$\frac{\pi}{4}$
$\frac{dA}{d\alpha}$	+ 0 -
A	

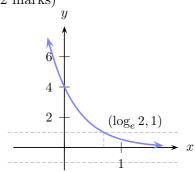
- $\alpha < \frac{\pi}{4}, \frac{dA}{d\alpha} < 0.$ $\alpha > \frac{\pi}{4}, \frac{dA}{d\alpha} > 0.$

Maximum area occurs when

$$A = 5000 \left(\sin \alpha + \cos \alpha \right) \Big|_{\alpha = \frac{\pi}{4}}$$

$$= 5000 \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) = 5000 \left(\frac{2}{\sqrt{2}} \right)$$

$$= 5000\sqrt{2} \text{ m}^2$$



$$(1 \text{ mark})$$

$$2e^{x} + 8e^{-x} + 32e^{-3x} \cdots$$

$$\frac{T_{2}}{T_{1}} = \frac{8e^{-x}}{2e^{x}} = 4e^{-2x}$$

$$\frac{T_{3}}{T_{2}} = \frac{32e^{-3x}}{8e^{-x}} = 4e^{-2x}$$

$$\frac{T_{2}}{T_{1}} = \frac{T_{3}}{T_{2}}$$

 $\therefore 2e^x + 8e^{-x} + 32e^{-3x} \cdots$ is a geometric series with $a = 2e^x$ and $r = 4e^{-2x}$.

 (β) (2 marks)

 \checkmark [1] for $|4e^{-2x}| < 1$.

 \checkmark [1] for justification.

A geometric series has a limiting sum when -1 < r < 1; i.e.

$$-1 < 4e^{-2x} < 1$$

By inspecting the graph in the previous part, $-1 < 4e^{-2x} < 1$ when $x > \log_e 2$.

: limiting sum exists when

$$x > \log_e 2$$

 (γ) (2 marks)

 \checkmark [1] for recalling formula

 \checkmark [1] for final answer

$$S = \frac{a}{1 - r} = \frac{2e^x}{1 - 4e^{-2x}}$$